HW 6 Help

32. ORGANIZE AND PLAN Like the previous problem, to obtain the frequency in Hz we need to know the orbital period in seconds. The orbital period of Jupiter is 11.9 years (which is $11.9 \text{ yrs} \cdot 365 \frac{\text{days}}{\text{yr}} = 4.34 \times 10^3 \text{days}$.

Converting days to seconds:

$$4.34\times10^3 \text{ days} \frac{\text{days}}{\text{osc}} \cdot \frac{3600 \text{ s}}{1 \text{ hr}} \cdot \frac{24 \text{ hr}}{\text{day}} = 3.75\times10^8 \frac{\text{s}}{\text{osc}}$$

With the period T in hand we obtain the frequency f in Hz: $f = T^{-1}$

SOLVE The frequency of the orbital motion of Jupiter around the Sun is:

$$f_J = T^{-1} = 2.67\times10^{-9} \text{ Hz}$$

REFLECT The frequency of the orbital motion of Jupiter is roughly an order of magnitude smaller than that of the Earth. In fact, $\frac{f_E}{f_J} = 11.9$.

38. ORGANIZE AND PLAN We are given the mass and spring constant in a mass-spring system. The period of a mass-spring system is given by $T = 2\pi\sqrt{\frac{m}{k}}$

The frequency (in oscillations per second Hz) is simply the inverse of the period: $f = 1/T$

The angular frequency ω (in units of radians per second) is obtained via a unit transformation of the frequency in Hz.

$$\omega = 2\pi\left[rad/\cancel{osc}\right]\cdot f\left[\cancel{osc}/s\right] = 2\pi f\left[rad/sec\right]$$

SOLVE Plugging in values: The period:

$$T = 2\pi\sqrt{\frac{0.450 \text{ kg}}{55.2 \text{ N/m}}} = 0.567 \text{ s}$$

The frequency:

$$f = 1.76 \text{ Hz}$$

The angular frequency:

$$\omega = 11.1 \text{ rad/s}$$

REFLECT Again, we demonstrate the utility of unit analysis. You must be able to comfortably transition between the period, frequency, and angular frequency.

41. ORGANIZE AND PLAN Given the bungee jumper follows the equation of motion:

$$x(t) = 5 \text{ m} \cos(\omega t)$$

we can find the position at any time as long as we know ω.
In the previous problem we are given the frequency $f = 0.125$ Hz. The relationship between the angular frequency has been derived above and in the text:

$$\omega = 2\pi f$$

So, in this problem $\omega = 2\pi 0.125$ rad/sec = 0.785 rad/sec

SOLVE Plugging in various values of time:

$$x(0.25 \text{ s}) = 5 \text{ m} \cos\ 0.785 \text{ rad/s}\ \ 0.25 \text{ s} = 4.9 \text{ m}$$
$$x(0.50 \text{ s}) = 5 \text{ m} \cos\ 0.785 \text{ rad/s}\ \ 0.50 \text{ s} = 4.6 \text{ m}$$
$$x(1.0 \text{ s}) = 5 \text{ m} \cos\ 0.785 \text{ rad/s}\ \ 1.0 \text{ s} = 3.5 \text{ m}$$

REFLECT With a frequency of oscillation of 0.125 Hz and corresponding period of 8 s there is not much action happening in the first second of motion. Referring to the figure in Problem 40 one second and less barely takes us out of the first hump in the oscillation. Our numerical answers jibe with the graphical results.

56. ORGANIZE AND PLAN The total mechanical energy in a mass-spring system is equal to the maximum instantaneous potential energy stored in the system as well as the maximum instantaneous kinetic energy. The maximum kinetic energy is $E = K_m = \frac{1}{2} m v_{max}^2$ We are given the mass and maximum velocity. The solution is simple insertion of values.

SOLVE The total energy is:

$$E = \frac{1}{2} 1.4 \text{ kg} \cdot (0.670 \text{ m/s})^2 = 0.314 \text{ Joules}$$

REFLECT A straightforward application of the conservation of energy and the knowledge obtained in Problem 50.

62. ORGANIZE AND PLAN We are given the mass, the total energy, and the amplitude of oscillation.

The total energy E of a mass-spring oscillator is equivalent to the maximum potential energy stored in the spring:

$$E = \frac{1}{2}kA^2$$

Isolating for k: $k = \frac{2E}{A^2}$

The period is given by: $T = 2\pi\sqrt{\frac{m}{k}}$

The maximum velocity is given by: $v_m = A\sqrt{\frac{k}{m}}$

The maximum acceleration is given by: $a_m = A\frac{k}{m}$

SOLVE The spring constant is $k = \frac{2 \cdot 125 \text{ J}}{(1.50 \text{ m})^2} = 111 \text{ N/m}$

The period is $T = 2\pi\sqrt{\frac{0.750 \text{ kg}}{111 \text{ N/m}}} = 0.516 \text{ s}$

The maximum velocity is given by: $v_m = 1.50 \text{ m}\sqrt{\frac{111 \text{ N/m}}{0.750 \text{ kg}}} = 18.2 \text{ m/s}$

The maximum acceleration is given by: $a_m = 1.50 \text{ m}\frac{111 \text{ N/m}}{0.750 \text{ kg}} = 222 \text{ m/s}^2$

REFLECT Compared to Problem 61 this oscillation is much faster, producing larger velocities and accelerations. The spring constant is much larger than in Problem 61 (111 compared to 4) and the mass is less than half. At first glance the large maximum acceleration seems incommensurate with the velocity of 18.2 m/s. However, when you consider that this acceleration must occur over a very short time scale given the period, it reflects the stiffness of the spring and the relatively low mass.

68. **ORGANIZE AND PLAN** The period of a simple pendulum in the small angle limit is $T = 2\pi\sqrt{\frac{L}{g}}$. Isolating for g yields:

$$g = \frac{4\pi^2 L}{T^2}$$

SOLVE Plugging in values: The gravitational constant is $g = \frac{4\pi^2 2.20\text{ m}}{(2.87\text{ s})^2} = 10.5\text{ m/s}^2$

REFLECT This acceleration is slightly larger than the acceleration due to gravity on the Earth. You would weigh more on this planet.

71. **ORGANIZE AND PLAN** If there are N oscillations over a duration Δt the period is time per oscillation or simply

T = $\frac{\Delta t}{N}$.

Given the period, we obtain the length by assuming $g = 9.8\text{ m/s}^2$ inverting the relationship for the period yielding:

$$L = \frac{gT^2}{4\pi^2}$$

SOLVE Plugging in values: The period is $T = \frac{32\text{ s}}{25\text{ osc}} = 1.28\text{ s}$

The length corresponding to this period is $L = \frac{9.8\text{ m/s}^2 \cdot (1.28\text{ s})^2}{4\pi^2} = 0.407\text{ s}$

REFLECT The roughly half a meter long pendulum with the 1.28 s period is consistent with experience.

79. **ORGANIZE AND PLAN** Criteria for light, critical or heavy damping is shown on Table 7.3. The values that must be compared are b^2 and 4 mk.

For the given values $b^2 = 7.02\text{ kg}^2/\text{s}^2$ and $4\text{ mk} = 480\text{ kg}^2/\text{s}^2$

SOLVE From calculated values we note that $b^2 < 4$ mk which corresponds to the lightly damped solution.

REFLECT If set into motion this oscillator will oscillate back and forth around the equilibrium value with ever decreasing amplitude.

80. **ORGANIZE AND PLAN** Criteria for light, critical or heavy damping is shown on Table 7.3. The values that must be compared are b^2 and 4 mk.

For the given values $b^2 = 146\text{ kg}^2/\text{s}^2$ and $4\text{ mk} = 40.7\text{ kg}^2/\text{s}^2$

SOLVE From calculated values we note that $b^2 > 4$ mk which corresponds to the heavily or over-damped solution.

REFLECT If set into motion this oscillator will not oscillate back and forth but slowly approach the equilibrium value from one direction.

96. **ORGANIZE AND PLAN** The period of oscillation of the subsequent motion after the bullet has struck and embedded itself in the block is $T = 2\pi\sqrt{\frac{M_T}{k}}$. The total mass of the bullet block system $M_T = M_B + m_b = 1.77$ kg.
The amplitude is obtained by determining the velocity the block-bullet system had immediately after the bullet has been embedded in the block. Using conservation of energy

$$\frac{1}{2}M_T v_{max}^2 = \frac{1}{2}kA^2$$

Isolating for A yields:

$$A = v_{max}\sqrt{M_T / k}$$

As in Problem 92, we must use conservation of momentum to find the velocity of the block bullet system after the bullet has been embedded. From derived result in 92 we obtain the velocity $v_{max} = \frac{m_b}{M_B + m_b}v_b$ where v_b is the velocity of the bullet before it hit the block. For this problem:

$$v_{max} = \frac{m_b}{M_B + m_b}v_b = \frac{0.0065\text{ kg}}{1.76\text{ kg} + 0.0065\text{ kg}}495\text{ m/s} = 1.82\text{ m/s}$$

The total energy of the whole system before the bullet strikes the block (E_b) is simply the kinetic energy of the bullet.
After the bullet hits the block the energy is $E_a = 1/2\,kA^2$.

SOLVE The period of oscillation of the mass-bullet system: $T = 2\pi\sqrt{\frac{1.77\text{ kg}}{85.0\text{ N/m}}} = 0.906$ s
The amplitude of the oscillations: $A = 1.82\text{ m/s}\sqrt{(1.77\text{ kg})/(85\text{ N/m})} = 0.263$ m
The energy before is $E_b = \frac{1}{2}\cdot(0.0065\text{ kg})\cdot(495\text{ m/s})^2 = 796$ J

The energy after is $E_a = \frac{1}{2}\cdot(85.0\text{ N/m})\cdot(0.263\text{ m})^2 = 2.93$ J

REFLECT Where did the energy go? The energy was lost in deforming the bullet as it entered the block, splitting the material of the block to let the bullet travel through, thermal energy in heating up the bullet. Rest assured, energy is still conserved. Kinetic energy, however, was converted into breaking atomic bonds and heating things up. Conservation theories are powerful things. In this problem we used two; 1 Conservation of momentum and 2 Conservation of energy.